Reg. No. : ……………………………………………

Name : ………………………………………………………………..

Second year Higher Secondary Examination

PART III

MATHEMATICS (SCIENCE)

Maximum: 80 (Scores)

TIME: $2\frac{1}{2}$ Hours

Cool-off time: 15 minutes

GENERAL INSTRUCTIONS TO CANDIDATES:

- There is a 'Cool-off time' of 15 minutes in addition to the writing time of $2\frac{1}{2}$ hours.
- You are not allowed to write your answers or to discuss anything with others during the 'Cool-off time'.
- Use 'Cool-off time' to get familiar with questions and to plan your answers.
- Read questions carefully before answering.
- All questions are compulsory and only internal choice is allowed.
- When you select a question, all the sub-questions must be answered from the same question itself.
- Calculations, figures and graphs should be shown in the answer sheet itself.
- Give equations wherever necessary.
- Electronic devices except non-programmable calculators are not allowed in the Examination Hall.

Questions 1 to 7 carry 3 score each. Answer any six.

1. a) Let $*$ be a binary operation, defined by $a*b=3a+4b-2$, find $4*5$ (1)

   b) Let $A=N\times N$ and $*$ be a binary operation on A defined by (a,b) $*$ (c,d)=(a+c,b+d). Show that $*$ is commutative and associative. Also, find the identity element for $*$ on A, if any. (2)

2. Solve $\tan^{-1}\left(\frac{x-1}{x-2}\right)+\tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}$ (3)

3. a) If the matrix A is both symmetric and skew-symmetric, then A is a ------------ matrix. (1)

   b) Find the inverse of $A=\begin{bmatrix}2 & -3\\ -1 & 2\end{bmatrix}$ by elementary row operation. (2)

4. Using properties of determinants, prove that

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ y+z & z+x & x+y \end{vmatrix}=(x-y)(y-z)(z-x)(x+y+z).$$ (3)

5. At what points will the tangent to the curve $y=2x^3-15x^2+36x-21$ be parallel to the x-axis? Also find the equations of the tangents to the curve at these points. (2)

6. Find a unit vector perpendicular to both the vectors $2\hat{i}+4\hat{j}-5\hat{k}$ *and* $\hat{i}+2\hat{j}+3\hat{k}$. (3)

7. Consider a vector $\vec{r}=2\hat{i}+3\hat{j}-6\hat{k}$.
   a) Find magnitude of $\vec{r}$ (1)
   b) Find the direction cosines of $\vec{r}$ (1)
   c) Show that $\sin^2\alpha+\sin^2\beta+\sin^2\gamma=2$ (1)

Questions 8 to 17 carry 4 score each. Answer any eight.

8. Consider the following system of equations:

$$x+y+z=6$$
$$x-y+z=2$$
$$2x+y+z=1$$

   i) Express this system of equations in the standard form $AX=B$. (1)
   ii) Prove that A is non-singular. (1)
   iii) Find the values of x,y and z satisfying the above system of equations. (2)

9. a) If $f(x)=x+7$ and $g(x)=x-7, x\in R$, find $(fog)(7)$. (1)

   b) Let $f:N\to N$ defined by $f(n)=\begin{cases}\frac{n+1}{2}, & if\ n\ is\ odd\\ \frac{n}{2}, & if\ n\ is\ odd\end{cases}$ for all $n\in N$. Find whether the function *f* is bijective. (2)

   c) Find the inverse of the function $f(x)=4x+3$ (1)

10. Find the value of a and b such that the function $f(x)=\begin{cases}5, & if\ \ x\le 2\\ ax+b, & if\ \ 2<x<10\\ 21, & if\ \ x\ge 10\end{cases}$

    is a continuous function. (4)

11. a) At the point x = 0, the function f (x) = |x| is

    (a) continuous, but not differentiable (b) differentiable, but not continuous

    (c) continuous and differentiable (d) neither continuous not differentiable (1)

    b) If $x\sin(a+y)=\sin y$, then prove that $\frac{dy}{dx}=\frac{\sin^2(a+y)}{\sin a}$. (3)

12. a) A spherical bubble is decreasing in volume at the of 2c.c/s. Find the rate at which the surface area is diminishing when the radius is 3 cm. (2)

    b) Find the equation of the tangent to the curve $y=x^2-4x+1$ at (2,3) (2)

13. Prove that $y=\frac{4\sin\theta}{(2+\cos\theta)}-\theta$ is an increasing function of θ in $\left[0,\frac{\pi}{2}\right]$. (4)

14. a) If $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$ prove that $\vec{a}$ and $\vec{b}$ are orthogonal. (2)

    b) Using vectors, Find x such that the points A(3,2,1), B (4, x ,5), C(4,2,-2) and D (6,5,-1) are coplanar. (2)

15. a) Find the equation of the plane passing through (2,–3,1) and is perpendicular to the line through the points (3,4,–1) and (2,–1,5). (2)

b) Find the distance from origin to the plane $3x-2y+6z+14=0$ (2)

16. Consider the lines $\frac{x-2}{2}=\frac{y-1}{5}=\frac{z+3}{-3}$ and $\frac{x+2}{-1}=\frac{y-4}{8}=\frac{z-5}{4}$.

a) Find the vector equations of the above lines. (2)

b) Find the angle between the lines. (2)

17. a) Find the sum of order and degree of differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}=5\frac{d^2y}{dx^2}$ (1)

b) Form the differential equation of the family of circles touching the y axis at origin. (3)

Questions 18 to 24 carry 6 score each. Answer any Five questions.

18. Show that the semi-vertical angle of a right circular cone of given surface area and maximum volume is $\sin^{-1}\left(\frac{1}{3}\right)$. (6)

19. a) If $\vec{\alpha}=3\hat{i}+4\hat{j}+5\hat{k}$ and $\vec{\beta}=2\hat{i}+\hat{j}-4\hat{k}$ then express $\vec{\beta}$ in the form $\vec{\beta}=\vec{\beta}_1+\vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$. (3)

b) For any three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ prove that $\left[\vec{a}+\vec{b}\quad \vec{b}+\vec{c}\quad \vec{c}+\vec{a}\right]=2\left[\vec{a}\ \vec{b}\ \vec{c}\right]$. (3)

20. Consider an L.P.P to minimize $z=3x+5y$ subject to the constraints:

$x+3y\geq 3,\ x+y\geq 2,\ x,y\geq 0$.

a) Draw the feasible region. (3)

b) Write the corner points of the feasible region. (1)

c) Find the minimum profit. (2)

21. Find the following integrals:

a) $\int \frac{\sqrt{\tan x}}{\sin x\cos x}dx$ (2)

b) $\int \frac{1}{x\left(x^4-1\right)}dx$ (2)

c) $\int x\tan^{-1}x\,dx$ (2)

22. a) Evaluate $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$ (1)

b) Evaluate: $\int_{-5}^{5}\left|x+2\right|dx$ (2)

c) Evaluate $\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}dx$ (3)

23. a) Consider the differential equation $\left(1+e^{\frac{x}{y}}\right)dx+e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy=0$

i) Show that it is a homogeneous differential equation. (1)

ii) Solve the above differential equation. (3)

b) Find the integrating factor of the differential equation: $x\frac{dy}{dx}+2y=x^2\log x$. (2)

24. Using integration find the area of the region bounded by the triangle whose vertices are $(-1,0)$, $(1,3)$ and $(3,2)$. (6)

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